Published by Institute of Physics Publishing for SISSA/ISAS
Received: *April 16, 2002*
Accepted: *April 30, 2002*

Higgs phenomenon for 4-d gravity in anti-de Sitter space

Massimo Porrati
Department of Physics, NYU
4 Washington Pl, New York NY 10003, U.S.A.
E-mail: massimo.porrati@nyu.edu

Abstract: We show that standard Einstein gravity coupled to a *free* conformal field theory (CFT) in anti-de Sitter space can undergo a Higgs phenomenon whereby the graviton acquires a nonzero mass (and three extra polarizations). We show that the essential ingredients of this mechanism are the discreteness of the energy spectrum in AdS space, and unusual boundary conditions on the elementary fields of the CFT. These boundary conditions can be interpreted as implying the existence of a 3-d defect CFT living at the boundary of AdS_4. Our free-field computation sheds light on the essential, model-independent features of AdS_4 that give rise to massive gravity.

Keywords: Spontaneous Symmetry Breaking, AdS-CFT and dS-CFT Correspondence, Space-Time Symmetries, Models of Quantum Gravity.

Contents

1. Introduction

Certain compactifications of 5-d gravity to 4 dimensions can be interpreted holographically as 4-d gravity coupled to matter. In this picture, the Kaluza-Klein excitations of the fifth dimension are bound states of the matter sector. Examples of models that admit a holographic interpretation are the Randall-Sundrum compactifications (RS) [1, 2] and the Karch-Randall compactification (KR) [3, 4]. The holographic interpretation of RS was spelled out in several papers [5, 6, 7], while that of KR was carried out in [8]. In [8] it was argued that KR is dual to a 4-d conformal field theory coupled to gravity on AdS_4. One of the most surprising aspects of KR is that it contains massive gravitons *only*. In the 4-d dual, the mass of the graviton arises from the graviton self-energy. In the holographic approximation, where graviton loops are neglected, this is the same as computing the 2-point function of the stress-energy tensor of the CFT. This computation was carried out in [8]. By giving a purely 4-d interpretation to KR, ref. [8] made clear that KR is the first example of a local 4-d field theory in which general covariance does not imply the existence of a massless graviton.[1] The dual of KR is gravity coupled to a *strongly self-interacting* CFT. Since the graviton mass does not come from the integrated conformal anomaly [8], it is not clear whether massive AdS_4 gravity is peculiar only to strongly interacting CFTs. To answer this question one should compute the graviton self-energy in a *weakly interacting* CFT. In [8], it was suggested to compute the graviton self-energy in the "least holographic" model available to us: a free conformal scalar.

[1] In 3-d a local modification of the Einstein lagrangian exists, that makes the graviton massive [9]. The modification is a Chern-Simons term that does not exist in even dimensions.

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In this paper we perform that calculation and we find that even when the CFT is free, the graviton can acquire a nonzero mass. Even more interestingly, we can pinpoint the property of the CFT that gives rise to massive gravity.

We begin our paper by reviewing in section 2 the consequences of general covariance and Weyl invariance on the two-point function of the stress-energy tensor (a.k.a. graviton self-energy). We show that the Ward identities due to those symmetries only constrain the self-energy to be transverse and traceless (tt). We also explain how a graviton mass shows up to quadratic order in the tt part of the self-energy. Finally, we explain the relation of our findings to the Stückelberg formalism.

In section 3 we restrict our analysis to free conformal theories in AdS_4. We show why a Higgs-like mechanism that makes the graviton massive can take place in free field theories in anti-de Sitter space, but not in Minkowsky space. Our analysis makes clear why the induced mass of the AdS_4 graviton is $O(\Lambda^2/M_{Pl}^2)$. We also analyze the role of boundary conditions and we find why graviton mass generation requires non-standard AdS_4 boundary conditions. Finally, we tentatively re-interpret those boundary conditions as due to the coupling of the 4-d bulk theory to a defect 3-d CFT.

Section 4 contains the explicit calculation of the graviton self-energy when the matter CFT is a single free, conformally coupled scalar. That calculation allows us to find the induced graviton mass.

Section 5 summarizes the findings in this paper, and it contains some concluding remarks; among them, a brief discussion of the model-independent features of our calculation, and comments on possible generalizations to bigravity models [4].

Our metric convention is "mostly plus," γ^0 is anti-hermitean, the γ^is are hermitean.

2. Ward identities and the Stückelberg mechanism

Let us consider a CFT on a 4-d anti-de Sitter space. Let $W[g]$ be the generating functional of the correlators of the stress-energy tensor. When expanded on an AdS_4 background, the one-point function $\delta W[g]/\delta g_{\mu\nu}(x) \equiv \langle T^{\mu\nu}\rangle$ does not vanish. Indeed, by denoting with $E_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ the Euler density, and with $C_{\mu\nu\rho\sigma}$ the Weyl tensor, we have

$$g_{\mu\nu}\frac{\delta W[g]}{\delta g_{\mu\nu}(x)} = aC_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + bE_4 + c\,\Box\, R\,. \tag{2.1}$$

Here a, b, c are constants that depend on the specific CFT. On an AdS background $C_{\mu\nu\rho\sigma} = \Box\, R = 0$ and eq. (2.1) together with the SO$(2,3)$ symmetry of the background implies

$$\left.\frac{\delta W[g]}{\delta g_{\mu\nu}(x)}\right|_{g=\bar{g}} = \frac{b}{24}\bar{g}^{\mu\nu}\bar{R}^2\,. \tag{2.2}$$

Background values for $g_{\mu\nu}$, R etc. will be denoted hereafter by an overbar. In terms of the AdS curvature radius L, we have $\bar{R} = 4\Lambda = -12/L^2$.

It is convenient to introduce now another quantity, $\hat{W}[g] = W[g] - (12b/L^4)\sqrt{g}$, that is stationary on the AdS background:

$$\left.\frac{\delta\hat{W}[g]}{\delta g_{\mu\nu}(x)}\right|_{g=\bar{g}} = 0\,. \tag{2.3}$$

If we couple our CFT to Einstein gravity, the two-point function of the stress-energy tensor *is* the matter contribution to the one-loop graviton self-energy, here called Σ

$$\Sigma^{\mu\nu,\rho\sigma}(x,y) \equiv \left.\frac{\delta^2\hat{W}[g]}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(y)}\right|_{g=\bar{g}}\,. \tag{2.4}$$

One may think that the Ward identities due to general diffeomorphisms and Weyl invariance would forbid a mass for the graviton, or, at least, relate it to the conformal anomaly, but this is not the case. To see this, we expand $\hat{W}[g]$ to quadratic order around the AdS background:

$$\hat{W}[g] = \hat{W}[\bar{g}] + \frac{1}{2}h_{\mu\nu} * \Sigma^{\mu\nu,\rho\sigma} * h_{\rho\sigma} + O(h^3), \qquad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\,. \tag{2.5}$$

Here, $A * B \equiv \int d^4x\sqrt{\bar{g}}A(x)B(x)$. The Ward identities of diffeomorphisms and local conformal (Weyl) invariance are eq. (2.1) and

$$D_{(\mu}\epsilon_{\nu)} * \frac{\delta W[g]}{\delta g_{\mu\nu}} = 0\,. \tag{2.6}$$

By using the expansion of $\hat{W}[g]$ given in eq. (2.5), the two Ward identities become, to linear order in the metric fluctuations,

$$D_{(\mu}\epsilon_{\nu)} * \Sigma^{\mu\nu,\rho\sigma} * h_{\rho\sigma} = 0\,, \qquad \bar{g}_{\mu\nu}\Sigma^{\mu\nu,\rho\sigma} * h_{\rho\sigma} = 0\,. \tag{2.7}$$

These identities simply state that Σ is transverse and traceless. They do not constrain at all the tt part of the self-energy.

They do not forbid a graviton mass either. Indeed, Ward identities never forbid a mass term for a gauge field. Let us consider, as a warm-up exercise, the simpler example of QED in Minkowsky space. In that case, the Ward identities of QED imply that the photon self-energy is transverse, i.e., in momentum space,

$$\Sigma_{\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)F(p^2)\,. \tag{2.8}$$

If $\lim_{p^2\to 0} F(p^2) \neq 0$ then the photon acquires a nonzero mass. Equivalently, we can see that mass is allowed by gauge invariance by using the Stückelberg formalism, i.e. by writing the lagrangian density of a massive spin-1 field as

$$L = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(\partial_\mu\phi - A_\mu)(\partial^\mu\phi - A^\mu)\,. \tag{2.9}$$

This lagrangian density is invariant under the gauge transformation $\phi \to \phi + \omega$, $A_\mu \to A_\mu + \partial_\mu\omega$. In the unitary gauge, $\phi = 0$, it reduces to the usual lagrangian density of a spin-1 field of mass m. If we integrate out the Sückelberg field ϕ, it reduces instead to a non-local action, that contributes to the self-energy Σ a term as in eq. (2.8), with $F(p^2) = m^2/2$. To sum up, neither Ward identities nor locality rule out a mass term.

In the case of a spin-2 field in AdS_4 the basic mechanism at work is the same as in our example, even though details differ. First of all, the projector over transverse-traceless states is more involved that in flat space. To find it, it is most convenient to introduce the Lichnerowicz differential operator Δ [10] On spin-2 fields, it reads

$$\Delta h_{\mu\nu} = -\Box h_{\mu\nu} - 2R_{\mu\rho\nu\sigma}h^{\rho\sigma} + 2R^{\rho}_{(\mu}h_{\nu)\rho}\,. \tag{2.10}$$

On the AdS background, $\bar{R}_{\mu\rho\nu\sigma} = (\Lambda/3)(\bar{g}_{\mu\nu}\bar{g}_{\rho\sigma} - \bar{g}_{\nu\rho}\bar{g}_{\mu\sigma})$, $\bar{R}_{\mu\nu} = \Lambda\bar{g}_{\mu\nu}$, and the Lichnerowicz operator obeys the following properties

$$\Delta D_{(\mu}V_{\nu)} = D_{(\mu}\Delta V_{\nu)}, \qquad \Delta V_\mu = (-\Box + \Lambda)V_\mu\,, \tag{2.11}$$

$$D^\mu \Delta h_{\mu\nu} = \Delta D^\mu h_{\mu\nu}\,, \tag{2.12}$$

$$\Delta \bar{g}_{\mu\nu}\phi = \bar{g}_{\mu\nu}\Delta\phi, \qquad \Delta\phi = -\Box\phi\,, \tag{2.13}$$

$$D^\mu \Delta V_\mu = \Delta D^\mu V_\mu\,. \tag{2.14}$$

These equations state that Δ commutes with covariant derivatives and trace. This is why in its definition we could omit the index labeling the degree of the form on which it acts. Let us consider now the rank-2 symmetric tensor $h_{\mu\nu}$. Its tt projection must have the form

$$\begin{aligned} h^{tt}_{\mu\nu} = \Pi_{\mu\nu}^{\rho\sigma} * h_{\rho\sigma} \equiv{}& A(\Delta)h_{\mu\nu} + B(\Delta)D_{(\mu}D^\lambda h_{\nu)\lambda} + C(\Delta)D_\mu D_\nu D^\lambda D^\rho h_{\lambda\rho} + \\ &+ D(\Delta)D_\mu D_\nu h + E(\Delta)\bar{g}_{\mu\nu}h + F(\Delta)\bar{g}_{\mu\nu}D^\rho D^\sigma h_{\rho\sigma}\,. \end{aligned} \tag{2.15}$$

Notice that, thanks to the properties of the Lichnerowicz operator given in eqs. (2.11)–(2.14) we can treat it as a number, as it commutes with covariant derivatives and traces. Using the properties of covariant derivatives on the AdS background, it is easy to see that tracelessness of $h^{tt}_{\mu\nu}$ implies two equations for the coefficients $A, \ldots, F$:

$$A + 4E - \Delta D = 0, \qquad B - \Delta C + 4F = 0\,. \tag{2.16}$$

Transversality, $D^\mu h^{tt}_{\mu\nu} = 0$, implies instead

$$2A + (2\Lambda - \Delta)B = 0\,, \qquad B + 2(\Lambda - \Delta)C + 2F = 0\,, \qquad E + (\Lambda - \Delta)D = 0\,. \tag{2.17}$$

In order to have a projection, $\Pi^2 = \Pi$, we must normalize $A = 1$. The other equations are then solved by

$$\begin{aligned} B &= \frac{2}{\Delta - 2\Lambda}\,, \qquad C = \frac{2}{(\Delta - 2\Lambda)(3\Delta - 4\Lambda)}\,, \qquad D = F = -\frac{1}{3\Delta - 4\Lambda}\,, \\ E &= \frac{\Lambda - \Delta}{3\Delta - 4\Lambda}\,. \end{aligned} \tag{2.18}$$

As we pointed out, Ward identities allow us to add a term proportional to Π to the two-point function of the stress-energy tensor

$$\Sigma^{\mu\nu\rho\sigma} * h_{\rho\sigma} = \frac{c}{2L^4}\Pi^{\mu\nu\rho\sigma} * h_{\rho\sigma} + \cdots\,. \tag{2.19}$$

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Here c is a dimensionless constant. The functional dependence on L is fixed simply by dimensional analysis, as the only scale appearing in Σ is the AdS curvature L. As in the spin-1 example given earlier, a nonzero c signals that the graviton acquires a nonzero mass. To see this, we couple the CFT to dynamical AdS gravity. Denote with $(16\pi G)^{-1}K$ (G=Newton's constant) the bare graviton kinetic term and integrate out the CFT. Assume that graviton loops can be neglected. The dressed kinetic term then becomes $(16\pi G)^{-1}K+\Sigma$, and the linearized equation of motion of the graviton is

$$[(16\pi G)^{-1}K_{\mu\nu}^{\ \ \rho\sigma}+\Sigma_{\mu\nu}^{\ \ \rho\sigma}]*h_{\rho\sigma}=0\,. \tag{2.20}$$

On tt fields $(Kh)^{tt}_{\mu\nu}=-(m^2/2)h^{tt}_{\mu\nu}$, and the equation of motion reduces to

$$\left[-(16\pi G)^{-1}m^2+\frac{c}{L^4}\right]h^{tt}_{\mu\nu}=0\,, \tag{2.21}$$

thus giving the value $16\pi Gc/L^4$ for the graviton square mass. Notice that if c is $O(1)$, the order of magnitude of the graviton mass is as in [3].

Notice also that the expression for Π contains the non-local term B, that can be interpreted as the propagator of a spin-1 Goldstone boson. Indeed, B has a pole whenever a transverse vector A_μ exists such that $(\Delta-2\Lambda)A_\mu=0$. In other words, the term proportional to B in eq. (2.15) plays for spin 2 on AdS the same role of the term $p^\mu p^\nu/p^2$ in eq. (2.8). In variance with Minkowsky space expectations, our Goldstone vector is *not* massless. In fact, a transverse, massless spin 1 in AdS_4 obeys the equation $\Delta A_\mu=0$, instead of $(\Delta-2\Lambda)A_\mu=0$. The latter equation states that the Goldstone vector is in a massive $\mathrm{SO}(2,3)$ representation. We will identify that representation in the next section; this one concludes by showing that the projection Π can be obtained from a local lagrangian by introducing a Stückelberg vector field, in exact parallel with the case of spin 1 in flat space. The Stückelberg vector A_μ is introduced into the linearized action of a massive spin-2 field (the Pauli-Fierz lagrangian [11]) by replacing everywhere the spin-2 field $h_{\mu\nu}$ with $h_{\mu\nu}+D_{(\mu}A_{\nu)}$. The Pauli-Fierz action on an AdS_4 background is then [12]

$$S=S_L[h_{\mu\nu}]+\int d^4x\sqrt{-\bar{g}}\,\frac{c}{4L^4}(h^2_{\mu\nu}-h^2)\,. \tag{2.22}$$

Here $S_L[h_{\mu\nu}]$ is the Einstein action with cosmological constant,

$$S_E[g_{\mu\nu}]=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R(g)-2\Lambda]\,,\qquad g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu} \tag{2.23}$$

linearized around the Einstein-space background $\bar{g}_{\mu\nu}$. The substitution $h_{\mu\nu}\to h_{\mu\nu}+D_{(\mu}A_{\nu)}$ maps S into

$$S_L[h_{\mu\nu}]+\int d^4x\sqrt{-\bar{g}}\,\frac{c}{4L^4}[(h_{\mu\nu}+D_{(\mu}A_{\nu)})^2-(h+2D_\mu A^\mu)^2]\,. \tag{2.24}$$

By integrating out the vector field A_μ, this action reduces to $S_L[h_{\mu\nu}]+(c/4L^4)h*\Pi*h$.

The action in eq. (2.24) is invariant under the linearized diffeomorphisms

$$h_{\mu\nu}\to h_{\mu\nu}+D_{(\mu}\epsilon_{\nu)}\,,\qquad A_\mu\to A_\mu-\epsilon_\mu\,. \tag{2.25}$$

It is far form evident that the Stückelberg mechanism can be made fully covariant; here, we limited ourselves to its linearization around a fixed background. The holographic interpretation of the KR model shows, on the other hand, that a complete covariantization of the Stückelberg mechanism is in fact possible. The same conclusion would follows if we were able to give a mass to the graviton by coupling Einstein gravity to a free CFT. The very possibility of this occurrence is due to some peculiar properties of AdS_4 that we analyze next.

3. SO(2, 3) representations and the gravitational Higgs mechanism in AdS_4

In the previous section we showed that diffeomorphism invariance does not forbid a mass term for the graviton. One cheap way to introduce such a mass would be to add by hand a term proportional to $\Pi^{\mu\nu\rho\sigma}$ into the graviton self-energy. This is the same as adding to the stress-energy tensor of matter a term $D_{(\mu}A_{\nu)}$, which is conserved if A_μ obeys the equation of motion $(\Delta - 2\Lambda)A^\mu = 0$. By thus changing the theory, we modify by hand its infrared behavior, by adding three extra degrees of freedom. Nothing guarantees us that this change make sense beyond the linearized level. Full consistency on the other hand *is* guaranteed if we find the extra degrees of freedom needed to give mass to the graviton in the stress-energy tensor of matter. In the case of a free field theory, where $T_{\mu\nu}$ is quadratic, this means finding the Goldstone vector as a bound state in the product of two free fields. In Minkowsky space this is obviously absurd since non-interacting two-particle states form a continuum. In anti-de Sitter space, instead, free particles do form bound states, since the AdS energy is quantized. To proceed further we need to review some facts about positive-energy representations of the AdS isometry group, SO(2, 3).

These representations were classified in [13] (see also [14] for a clear review). In the decomposition $SO(2,3) \to SO(2) \times SO(3)$, the generator of SO(2) is the AdS_4 energy, while angular momentum is given by the generators of SO(3). A unitary, irreducible, positive-weight representation of SO(2, 3) (UIR), $D(E, s)$, is labeled by the energy E/L and spin s of its (unique) lowest-energy state. E is the energy measured in units of the AdS curvature radius L. Free fields form irreducible representations of SO(2, 3). A conformal scalar can belong to either the $D(1, 0)$ or the $D(2, 0)$. A conformal (massless) spin-1/2 fermion belongs to a $D(3/2, 1/2)^\pm$, while a massless vector (also conformal) belongs to a $D(2, 1)^\pm$ [15, 16]. The label $\pm$ denotes the two possible parities of the UIR.

Massless representations of spin $s > 0$ have $E = s + 1$ [15, 16, 14]. Massive unitary representations of spin larger than zero have $E > s + 1$. In the limit $E \to s + 1$, the UIR $D(E, s)$, $s \geq 1$ becomes reducible [13, 15]:

$$D(E, s) \to D(s + 1, s) \oplus D(s + 2, s - 1)\,, \qquad E \to s + 1\,. \tag{3.1}$$

Eq. (3.1) encodes the group theoretical aspect of the Higgs phenomenon in AdS_4: when a spin-s field, $s \geq 1$, becomes massive, it "eats" a spin-$(s - 1)$ boson. Notice than for

$s = 0$ this boson is in a $D(3,0)$, i.e. it is a minimally-coupled scalar [16]. For spin 2, it is a *massive* vector in the $D(4,1)$.[2] Notice that the wave equation obeyed by a vector A_μ in the $D(4,1)$ is exactly what we found in the previous section: $(\Delta - 2\Lambda)A_\mu = 0$.

The next question we have to address is whether a $D(4,1)$ appears in the stress-energy tensor of a free CFT. Since the stress-energy tensor of a free field theory is quadratic in the fields, the $D(4,1)$ can only be in $T_{\mu\nu}$ if it appears in the tensor product of the UIRs to which the fields belong. Let us examine separately free conformally-coupled fields of spin 1, 1/2, and 0.

spin 1 Massless spin-1 fields are conformal; they belong to the $D(2,1)$ [15, 16]. The tensor product of SO(2,3) UIRs was found by Heidenreich in [17]. For $D(2,1)$, he found

$$D(2,1) \otimes D'(2,1) = \sum_{n=0}^{\infty} D(4+n,0) \oplus \sum_{n=0}^{\infty} D(4+n,1) \oplus \oplus \sum_{S=0}^{\infty} \Big[D(4+S, 2+S) \oplus \sum_{n=0}^{\infty} 2D(5+S+n, 2+S) \Big] . \tag{3.2}$$

In our case, since we are tensoring two identical bosons, some of the representations that appear in the tensor product above are absent. For instance, the ground state of the $D(4,1)$ that appears in the tensor product above is antisymmetric in its arguments $(\sim \epsilon_{ijk} A^i A^j)$, so it is forbidden by Bose statistics. This means that the entire $D(4,1)$ is absent.

spin 1/2 The massless (conformal) spin-1/2 field belongs to the $D(3/2, 1/2)$. Tensoring two different $D(3/2, 1/2)$, Heidenreich finds [17]

$$D\Big(\frac{3}{2},\frac{1}{2}\Big) \otimes D'\Big(\frac{3}{2},\frac{1}{2}\Big) = \sum_{n=0}^{\infty} D(3+n,0) \oplus \sum_{S=0}^{\infty} \Big[D(3+S, 1+S) \oplus \oplus \sum_{n=0}^{\infty} 2D(4+S+n, 1+S) \Big] . \tag{3.3}$$

In this tensor product, the $D(4,1)$ appears twice. By taking into account Fermi statistics when tensoring two identical representations, we get rid of one of them. The other one cannot appear in the stress-energy tensor since it has the wrong parity. To arrive at this result we first notice that the stress-energy tensor of a free CFT made of several fields of spin $s \leq 1$ is given by the sum $T_{\mu\nu} = \sum_i T^i_{\mu\nu}$. The i-component of this sum is the stress-energy tensor of either a real vector, a real scalar, or a Majorana fermion. To preserve the Majorana condition ($\psi = C\psi^*$, C = charge conjugation), the field $\psi(x)$ must transforms as follows under parity: $\psi(t, \mathbf{x}) \to \eta\gamma^0\psi(t, -\mathbf{x})$, $\eta = \pm 1$. The fermion field ψ can be expanded in spherical waves. Its positive-frequency part (with respect to the global AdS time t) is [16]

$$\psi_{pf} = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} e^{-i\omega t/L} a_{\omega j m} \chi^{\pm}_{\omega j m} , \qquad j = \frac{1}{2} + k , \qquad \omega = 1 + j + n . \tag{3.4}$$

[2]Recall that in Minkowsky space, instead, the Higgs phenomenon for a spin 2 requires a massless vector and a massless scalar. They together provide 3 degrees of freedom, as it does our massive vector in AdS_4.

The operators $a^\dagger_{\omega jm}, a_{\omega jm}$ respectively create and annihilate states of definite energy ω/L and angular momentum j, belonging to the $D(3/2,1/2)^\pm$. The superscript $\pm$ labels the parity of the representation. More precisely, when ψ belongs to the $D(3/2,1/2)^+$, the spherical waves $\chi^+_{\omega jm}$ in eq. (3.4) transforms under parity as $\chi^+_{\omega jm} \rightarrow i(-)^{\omega-3/2}\chi^+_{\omega jm}$. Analogously, for $D(3/2,1/2)^-$, $\chi^-_{\omega jm} \rightarrow -i(-)^{\omega-3/2}\chi^-_{\omega jm}$. The parity a UIR is fixed by the parity of its ground state. The assignments given above show immediately that the parity of the ground state of the $D(4,1)$ in the tensor product of either $D(3/2,1/2)^+ \otimes D(3/2,1/2)^+$ or $D(3/2,1/2)^- \otimes D(3/2,1/2)^-$ is $+1$. This is the parity of a *pseudo-vector*, while the $D(4,1)$ contained in $T_{\mu\nu}$ must be a true vector, with parity -1. This can be seen most easily by noticing that $T_{\mu\nu}$ is a true tensor and that the $D(4,1)$ we are after must appear in it as follows

$$T_{\mu\nu} = D_{(\mu}A_{\nu)} + \cdots, \qquad (\Delta - 2\Lambda)A_\mu = 0\,. \tag{3.5}$$

Equivalently, we may notice that with a single Majorana fermion we cannot form a vector, as $\bar\psi\gamma^\mu\psi = 0$, but we can form the pseudo-vector $\bar\psi\gamma^\mu\gamma^5\psi$.

spin 0 Scalars belong to $D(E,0)$, $E \geq 1/2$. The tensor product of two spin zero representations of $\mathrm{SO}(2,3)$ is [17]

$$D(E_1,0) \otimes D(E_2,0) = \sum_{S=0}^{\infty}\sum_{n=0}^{\infty} D(E_1+E_2+S+2n, S)\,. \tag{3.6}$$

Here $E_1, E_2 > 1/2$. When $E = 1/2$, the representation degenerates, becoming a singleton [18, 19], namely a representation that propagates only boundary degrees of freedom and cannot be represented as a standard local field living in the bulk of AdS_4. We will not consider it further. When $E_1 = E_2 = E > 1/2$, a $D(4,1)$ exists in the tensor product $D(E,0) \otimes D'(E,0)$ only for $E = 3/2$. If the two representations are identical, $D(4,1)$ is eliminated by Bose statistics (its would be ground state is in reality a descendant belonging to the $D(3,0)$).

This is not the end of the story, since the $D(4,1)$ appears in the tensor product of two representations of different energy. In particular, as it is evident from eq. (3.6), it appears in the product $D(1,0) \otimes D(2,0)$. As we mentioned earlier, a conformal scalar belongs to either $D(1,0)$ or $D(2,0)$. We can obtain a $D(4,1)$ by taking two different conformal scalars, A, B, one belonging to $D(1,0)$, the other to $D(2,0)$. Equivalently, we can form a vector out of the two scalars: $A_\mu \sim AD_\mu B$. This is not what we need though, as the stress-energy tensor does not mix the field A with B. What we need is more unconventional.

When we stated that a conformal scalar belongs to either a $D(1,0)$ or a $D(2,0)$ we implicitly assumed certain conditions at the boundary of AdS_4. These boundary conditions are spelled out [16]; they amount to ask that no momentum or energy escape through the boundary. We will refer to them as reflecting boundary conditions. Recent studies of of the KR model [3, 20, 8, 21, 22] have made clear that these boundary conditions are not the most general physically meaningful ones. For instance, the holographic interpretation of the KR model *demands* that energy and momentum freely pass through the AdS_4 boundary into a "mirror" AdS_4 space [22]. Equivalently, energy and momentum are absorbed and released into the 4-d bulk by a 3-d CFT living on the boundary of AdS_4. For us, this means that we can relax the boundary conditions of [16], i.e. that we can allow the conformal scalar to belong to the reducible representation $D(1,0) \oplus D(2,0)$. By doing this, the stress-energy tensor does contain fields in the tensor product $D(1,0) \otimes D(2,0)$, and it may even contain a $D(4,1)$.

Whether the $D(4,1)$ is really in $T_{\mu\nu}$ can only be found by an explicit calculation of the self-energy. This is the subject of the next section.

4. Through a glass, clearly: KR boundary conditions and the graviton self-energy

4.1 KR boundary conditions

We call KR the boundary conditions that allow a conformal scalar to freely pass through the boundary of AdS_4 into a mirror space obtained as follows: consider the static Einstein Universe, which is topologically $S_3 \times R$. By a Weyl rescaling $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, the Einstein universe is mapped into two adjacent AdS_4 spaces, joined at their common boundary. If ϕ solves the equations of motion of a conformal scalar in the Einstein-Universe background, then $\Omega^{-1}\phi$ solves the equations of motion of the scalar in AdS_4. A complete set of solutions in the Einstein Universe gives *two* complete sets of solutions in AdS_4, namely, $D(1,0)$ and $D(2,0)$. If we want to describe a scalar free to move from one AdS into the other, as required for instance for the holographic interpretation of the KR model, we must keep *both* sets of modes. More general boundary conditions can be imposed if we put a 3-d defect CFT at the boundary of AdS_4.

4.2 The scalar propagator

Consider R^5 with pseudo-euclidean metric $\eta = diag(-1,-1,+1,+1,+1)$. anti-de Sitter space is the covering space of the hyperboloid $X^M X^N \eta_{MN} = -L^2$, $M,N = 0,\dots,4$. By SO$(2,3)$ invariance, the scalar propagator $\Delta_E(X,Y)$ is a function of $Z \equiv X^M Y_M/L^2$ only. $\Delta_E(Z)$ obeys the equation

$$[(1-Z^2)\partial_Z^2 - 3Z\partial_Z + E(E-3)]\Delta_E(Z) = 0\,; \tag{4.1}$$

The mass of the scalar is related to E by the equation $L^2m^2 = E(E-3)$. The solution of equation (4.1) that vanishes at large Z is a hypergeometric

$$\Delta_E(Z) = rZ^{-E}F\left(E, E-1; 2E-2; \frac{1}{Z}\right). \tag{4.2}$$

The normalization constant r is fixed by requiring that at $Z \to 1$ $\Delta_E(Z)$ reduces to the properly normalized flat-space Green's function. As shown in [23], this condition gives

$$r = \frac{1}{4\pi^2 L^2}\frac{\Gamma(E)\Gamma(E-1)}{\Gamma(2E-2)}\,. \tag{4.3}$$

For conformal coupling, $E = 1$ or 2, and the normalized solution of eq. (4.1) reduces to

$$\Delta(Z) = \frac{1}{4\pi^2 L^2}\left(\alpha\frac{1}{Z^2-1} + \beta\frac{Z}{Z^2-1}\right). \tag{4.4}$$

When the scalar is in the $D(1,0)$, $\alpha = 0, \beta = 1$; when it is in the $D(2,0)$, $\alpha = 1, \beta = 0$. KR boundary condition give instead $\alpha = \beta = 1/2$. This can be seen as follows. The scalar propagator in any space is

$$\Delta(x,y) = \sum_i \frac{1}{\lambda_i}\phi_i(x)\phi_i(y)\,, \qquad (-\Box + m^2)\phi_i(x) = \lambda_i\phi_i(x)\,. \tag{4.5}$$

Call ψ_i the eigenmodes of eq. (4.5) that form a $D(1,0)$, and χ_i the eigenmodes that form the $D(2,0)$. They can be thought as the modes of $S_3 \times R$ restricted to a half 3-sphere. A complete set of normalized modes on S_3 is $\{2^{-1/2}\psi_i, 2^{-1/2}\chi_i\}$,[3] so the propagator on $S_3 \times R$ is

$$\begin{gathered}\Delta(x,y) = \frac{1}{2}\sum_i \frac{1}{\lambda_i}\psi_i(x)\psi_i(y) + \frac{1}{2}\sum_i \frac{1}{\lambda'_i}\chi_i(x)\chi_i(y)\,, \\ \left(\Box + \frac{2}{L^2}\right)\psi_i(x) = -\lambda_i\psi_i(x), \qquad \left(\Box + \frac{2}{L^2}\right)\chi_i(x) = -\lambda'_i\chi_i(x)\,. \end{gathered} \tag{4.6}$$

When restricted to AdS_4, eq. (4.6) reduces to eq. (4.4) with $\alpha = \beta = 1/2$. We will keep α, β generic in most of our calculations.

4.3 Tensor fields in homogeneous coordinates

We could compute the graviton self-energy in intrinsic coordinates, using the techniques of ref. [24, 23], but it is much simpler to use the embedding of AdS_4 in R^5 given in the previous subsection, and to promote all 4-d fields into 5-d homogeneous fields. That technique was developed in ref. [15]. Consider first a spin-2 field of mass $L^2 m^2 \equiv E(E-3)$, represented by a symmetric tensor $h^{\mu\nu}(x)$. The embedding of AdS_4 into R^5 defines 5 coordinates $X^M(x)$ obeying $X^M(x)X_M(x) = -L^2$. The 5-d tensor field $h^{MN}(X)$ is then defined as the homogeneous field of degree N that on the hyperboloid reduces to

$$h^{MN}(x) = \partial_\mu X^M(x)\partial_\nu X^N(x) h^{\mu\nu}(x)\,. \tag{4.7}$$

By construction it obeys

$$X^N\partial_N h^{AB}(X) = N h^{AB}(X)\,, \qquad X^M h_{MN}(X) = 0\,. \tag{4.8}$$

The 5-d indexes, $M.N$ etc. are raised and lowered with the flat metric η_{MN}, the degree of homogeneity N is arbitrary.

[3] The factor $2^{-1/2}$ ensures that the modes on S_3 are normalized to one when the modes on the half-sphere have unit norm, since ψ_i, χ_i are either symmetric or antisymmetric under the reflection that maps one hemisphere into the other.

As shown in [15], the 4-d equations of motion are equivalent to the following 5-d equations ($\partial^2 = \partial_M \partial^M$):

$$[X^2\partial^2 - (N+E)(N-E+3)]h_{MN} = 0\,, \qquad \partial_M h^{MN} = 0\,, \qquad h^M_M = 0\,. \tag{4.9}$$

Equivalently, the space of fields that solves eqs. (4.9) is $D(E,2)$ for $E > 3$. For $E = 3$, $h_{MN} = X^2\partial_{(M}A_{N)} + (2-N)X_{(M}A_{N)}$ also solves eqs. (4.9) when A_M obeys ($A\cdot B \equiv A^M B_M$)

$$[X^2\partial^2 - (X\cdot\partial)^2 - 3X\cdot\partial + 4]A_M = 0\,, \quad X\cdot A = 0\,, \quad \partial\cdot A = 0\,, \quad X\cdot\partial A_M = (N-1)A_M\,. \tag{4.10}$$

A_M is the gauge mode, generating the $D(4,1)$. This follows from the very definition of gauge mode. More generally, recall that a spin-1 field belongs to the $D(E,1)$ ($E \geq 2$). One can then show [15] that its 5-d equations of motion are

$$[X^2\partial^2 - (X\cdot\partial)^2 - 3X\cdot\partial + E(E-3)]V_M = 0\,, \qquad X\cdot V = 0\,, \qquad \partial\cdot V = 0\,. \tag{4.11}$$

These equations show once more that the gauge mode belongs to the $D(4,1)$.

Eqs. (4.10) allow us to find the operator P that projects the symmetric tensor h_{MN} over tt modes, i.e. the 5-d equivalent of the operator $\Pi^{\mu\nu\rho\sigma}$ given in eqs. (2.15), (2.18). The projector P decomposes as $P = I + P_1 + \sum_i P_0^i$. I is the identity operator on symmetric tensors, P_1 is the projector over spin-1 states, and the P_0^i are projectors over spin-0 states. We are interested in finding P_1, as it is the term that we will need to detect the presence of a Goldstone vector in the graviton self-energy. To find P_1, we can compute the scalar product of P in between symmetric tensors that are not only X-transverse, $X^M h_{MN} = 0$, but also traceless and double divergenceless:

$$h^M_M = \partial_M\partial_N h^{MN} = 0\,. \tag{4.12}$$

On these fields the projector simplifies considerably. Explicitly,

$$P^{CD}_{AB}h_{CD} = h_{AB} - 2[X^2\partial_{(A}\eta_{B)} + (2-N)X_{(A}\eta_{B)}] + \cdots\,, \tag{4.13}$$

$$\eta_A = [X^2\partial^2 - (X\cdot\partial)^2 - 3X\cdot\partial + 4]^{-1}(\partial^L h_{LA} + \cdots)\,. \tag{4.14}$$

The ellipsis denote terms that vanish in the scalar product. To define the scalar product $h * P * h$ we have to promote the 4-d integration oven AdS_4 to 5-d. This is done by promoting 4-d scalars $S(x)$ to homogeneous 5-d scalars of arbitrary degree N

$$X\cdot\partial\hat{S}(X) = N\hat{S}(X)\,, \qquad \hat{S}(X)\Big|_{X^2=-L^2} = S(x)\,. \tag{4.15}$$

The 4-d integration is then extended by using the identity

$$\int d^4x\sqrt{g}S(x) = \int d^5X\delta(X^2+L^2)\hat{S}(X) \equiv \int d\mu(X)\hat{S}(X)\,. \tag{4.16}$$

The scalar product $h * P * h$ on fields obeying eq. (4.12) finally reads

$$h * P * h = \int d\mu\Big\{h^{AB}h_{AB} + 2X^2\partial_C h^{CA}[X^2\partial^2 - (X\cdot\partial)^2 - 3X\cdot\partial + 4]^{-1}\partial^D h_{DA}\Big\}\,. \tag{4.17}$$

This equation identifies the projector over the spin-1 state: it is the term proportional to $(\partial\cdot h)^2$.

We need one last property before we embark in the computation of the graviton self-energy, namely the matrix element

$$\langle X|[X^2\partial^2 - (X\cdot\partial)^2 - 3X\cdot\partial + 4]^{-1}|Y\rangle\,. \tag{4.18}$$

When the two points X, Y lie on the hyperboloid $Y^2 = X^2 = -L^2$, the matrix element is a function of $Z \equiv X\cdot Y/L^2$ only. It obeys eq. (4.1) with $E = 4$ so it is equal to $-\Delta_4(Z)$. For $Z \to \infty$ eq. (4.3) gives

$$\Delta_4(Z) = \frac{1}{40\pi^2 L^2} Z^{-4} + O(Z^{-6}), \qquad Z \to \infty. \tag{4.19}$$

4.4 The graviton self-energy

We finally come to the heart of this paper: the computation of the graviton self-energy due to a free conformally coupled scalar. Since we neglect graviton loops in our computation, the only contribution to the self-energy comes from the 2-point correlator of the matter stress-energy tensor

$$h * \Sigma * h = \int d^4x\sqrt{\bar{g}(x)} \int d^4y\sqrt{\bar{g}(y)} h^{\mu\nu}(x)\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle h^{\rho\sigma}(y)\,. \tag{4.20}$$

A free conformal scalar on AdS_4 obeys the equation of motion $(\Box - 2\Lambda/3)\phi = 0$. Its stress-energy tensor is

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\bar{g}_{\mu\nu}\partial_\lambda\phi D^\lambda\phi - \frac{1}{6}[D_\mu D_\nu - \bar{g}_{\mu\nu}(\Box - \Lambda)]\phi^2\,. \tag{4.21}$$

We can simplify the calculation of the self-energy by evaluating eq. (4.20) on traceless, double divergenceless tensors

$$h^\mu_\mu = D_\mu D_\nu h^{\mu\nu} = 0\,. \tag{4.22}$$

On these configurations, eq. (4.20) becomes

$$h * \Sigma * h = \int d^4x\sqrt{\bar{g}(x)} \int d^4y\sqrt{\bar{g}(y)} h^{\mu\nu}(x)\langle \partial_\mu\phi(x)\partial_\nu\phi(x)\partial_\rho\phi(y)\partial_\sigma\phi(y)\rangle h^{\rho\sigma}(y)\,. \tag{4.23}$$

Since ϕ is a free field, we use Wick's theorem and eq. (4.4) to find

$$h * \Sigma * h = 2\int d^4x\sqrt{\bar{g}(x)} \int d^4y\sqrt{\bar{g}(y)} h^{\mu\nu}(x)h^{\rho\sigma}(y)\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\rho}\Delta(Z)\frac{\partial}{\partial x^\nu}\frac{\partial}{\partial y^\sigma}\Delta(Z)\,. \tag{4.24}$$

Now we convert eq. (4.24) into a 5-d equation using the results of the previous subsection. Namely, we define a 5-d field h_{MN} by eq. (4.7) and we transform the 4-d integration into a 5-d one using eq. (4.16). Recalling the definition of the coordinate $Z = X\cdot Y/L^2$, and setting $L = 1$ henceforth, we arrive at

$$\begin{aligned} h * \Sigma * h \,=\, & 2\int d\mu(X)\int d\mu(Y) h^{AB}(X)h^{CD}(Y)[\Delta'(Z)\eta^{AC} + Y^A X^C\Delta''(Z)] \times \\ & \times\, [\Delta'(Z)\eta^{BD} + Y^B X^D\Delta''(Z)]\,, \qquad {}' \equiv \frac{d}{dZ}\,. \end{aligned} \tag{4.25}$$

As previously explained, we want to see if eq. (4.25) contains a term proportional to $\partial \cdot h \Delta_4(Z) \partial \cdot h$. To see that, we first write eq. (4.25) as a sum of three pieces

$$h * \Sigma * h = 2 \int d\mu(X) \int d\mu(Y)(A + B + C) , \tag{4.26}$$

$$A = h^{AB}(X) h_{AB}(X) [\Delta'(Z)]^2 , \tag{4.27}$$

$$B = h_{AB}(X) h^{BD} Y^A \frac{\partial}{\partial Y^D} [\Delta'(Z)]^2 , \tag{4.28}$$

$$C = h^{AB}(X) h_{CD}(Y) X^C X^D \frac{\partial Z}{\partial X^A} \Delta''(Z) \frac{\partial}{\partial X^B} \Delta'(Z) . \tag{4.29}$$

By integrating by part repeatedly the functions B and C, we cast eq. (4.25) into the desired form

$$h * \Sigma * h = 2 \int d\mu(X) \int d\mu(Y) \frac{\partial}{\partial X^A} h^{AB}(X) \frac{\partial}{\partial Y^C} h^C_B(Y) [F(Z) - 4G(Z)] + \cdots , \tag{4.30}$$

$$F'(Z) = [\Delta'(Z)]^2 , \qquad G'''(z) = [\Delta''(Z)]^2 . \tag{4.31}$$

In this equation, we omitted all terms not proportional to $(\partial_A h^{AB})(X)(\partial^C h_{CB})(Y)$.

Now we can find if the stress-energy tensor of our theory contains a vector in the $D(4,1)$. We just need to find if $F - 4G$ contains a term proportional to the propagator of the vector. That propagator decays as Z^{-4} at large Z (see eq. (4.19)), so we need to find if such a term is contained in $F - 4G$. The constants of integration in the definition of F and G must be chosen so that they decay at large Z. If we recall the definition of $\Delta(Z)$ given in eq. (4.4) and we expand this function in powers of $1/Z$ we find that there is only one term proportional to Z^{-4}:

$$F - 4K = -\frac{1}{5(4\pi^2)^2} \alpha\beta Z^{-4} + \cdots . \tag{4.32}$$

Notice that this term vanishes when the b.c. are purely $E = 1$ or $E = 2$, as it should. Eq. (4.32) shows that mixed boundary conditions, with both α and β nonzero, do indeed give rise to a Goldstone vector. This happens in particular with the KR conditions, $\alpha = \beta = 1/2$. Upon coupling to gravity, this vector supplies the extra polarizations needed to make the graviton massive. To find the graviton mass induced by this Higgs mechanism, we first recall that the large-Z behavior of the propagator $\Delta_4(Z)$ is given by eq. (4.19), thus

$$F - 4K = -\frac{1}{2\pi^2} \alpha\beta \Delta_4(Z) + \cdots . \tag{4.33}$$

Finally, we re-introduce L in our formulas, recall eqs. (2.19), (2.20), and find:

$$m^2 = 16\pi G \frac{2\alpha\beta}{\pi^2 L^4} . \tag{4.34}$$

As predicted by dimensional analysis in section 2, the graviton mass is proportional to L^{-4} i.e. to Λ^2. We emphasize again that the functional dependence on L is model-independent, as it follows only from two simple assumptions: 1) graviton loops can be neglected in the computation of the graviton self-energy; 2) matter is conformal. The first assumption

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guarantees that the self-energy is independent of G; the second guarantees that L is the only mass scale of the matter theory. Clearly, the numerical coefficient in eq. (4.34) *is* model dependent already in the free theory, since it depends on the boundary conditions! Even when the boundary conditions are chosen to be KR ($\alpha = \beta = 1/2$), no non-renormalization theorem is known to us that protects eq. (4.34). It is not surprising, therefore, that the proportionality constant in eq. (4.34) does not coincide with what was found in the KR model [3, 8]. After all, KR is dual to a *strongly interacting* CFT, while here we examined a *free* CFT.

5. Conclusions

In this paper we re-examined the possibility of giving a mass to the graviton in anti-de Sitter space.

We pointed out that Ward identities do not forbid a graviton mass, at least in linearized gravity. We then proceeded to examine the conditions that allow a gravitational Higgs mechanism in AdS, and we gave a model-independent estimate of the graviton mass induced by a CFT.

Next, we recalled that unlike Minkowsky space, AdS_4 allows even a free theory to form bound states, owing to the discreteness of the AdS energy spectrum. We investigated free CFTs with spin not greater than 1. We found that, in order to have a Goldstone particle in the stress-energy tensor, we had to impose non-standard (i.e. non-reflecting) boundary conditions on the fields of our free CFT. Similar boundary conditions are not only allowed, but indeed necessary, to interpret holographically the KR model [3, 8, 20, 22].

We considered next a free conformal scalar in AdS_4, and we found that, even in that very simple example, nonstandard boundary conditions did produce a Goldstone boson that gives a mass to the graviton, when the CFT is coupled to gravity.

The present calculation found that a graviton mass is generated by coupling standard gravity to a free CFT. In [3, 8], it was shown that the same phenomenon happens when gravity is coupled to a *strongly* interacting CFT. In both cases the key ingredient is the boundary conditions imposed on the CFT. They must allow energy and momentum to flow in and out of AdS_4 through its boundary. This phenomenon can be interpreted as due to a 3-d defect conformal field theory located at the boundary of AdS_4. It is curious that two vastly different cases — free CFT versus strongly-interacting CFT — in which the graviton becomes massive have in common just the choice of boundary conditions. We may speculate that this fact point out to the possibility of finding a model-independent setting for the gravitational Higgs effect on AdS, that only depends on a choice of boundary dynamics for the field theory of matter.

Finally, we may ask if the "bigravity" model of ref. [4] can also be explained purely in terms of 4-d physics with nonstandard boundary conditions. We recall that in [4] a region of AdS_5 was bounded by *two* AdS_4 branes. In that model one finds, besides the usual massless graviton, a second spin-2 field that couples as the graviton and has mass $O(\Lambda^2/M_{pl}^2)$. When one of the two branes is sent to infinity, the coupling of the massless graviton vanishes while that of the massive graviton remains finite. In that limit, the model

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becomes KR. Clearly, the spectrum of bigravity cannot be reproduced by *any* choice of the parameters α, β in eq. (4.4), as long as they are constant. Nevertheless we suggest that a more complicated choice of boundary conditions, where α, β become functions of the AdS_4 energy, may give rise to bigravity. This choice of boundary conditions can be thought of as describing the dynamics of the 3-d defect CFT living at the boundary of AdS_4, that must partially reflect and partially absorb bulk fields.

Acknowledgments

We would like to thank N.N. Khuri for useful conversations. This work is supported in part by NSF grant PHY-0070787.

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